PROOF OF FORMULA 4.252.2

$$\int_0^\infty \frac{x^{\mu-1}\,\ln x}{(x+b)(x-1)}\,dx = \frac{\pi}{(1+b)(\sin\pi\mu)^2}\left[\pi - b^{\mu-1}(\sin\pi\mu\ln b - \pi\cos\pi\mu)\right]$$

The partial fraction decomposition

$$\frac{1}{(x+b)(x-1)} = \frac{1}{b+1}\frac{1}{x-1} - \frac{1}{b+1}\frac{1}{x+b}$$

yields

$$\int_0^\infty \frac{x^{\mu-1}\,\ln x}{(x+b)(x-1)}\,dx = \frac{1}{b+1}\int_0^\infty \frac{x^{\mu-1}\,\ln x}{x-1}\,dx - \frac{1}{b+1}\int_0^\infty \frac{x^{\mu-1}\,\ln x}{x+b}\,dx.$$

The result now follows by using 4.251.1

$$\int_0^\infty \frac{x^{\mu-1}\,\ln x}{x+b}\,dx = \frac{\pi b^{\mu-1}}{\sin\pi\mu}(\ln b - \pi\cot\pi\mu)$$

and 4.251.2

$$\int_0^\infty \frac{x^{\mu-1}\,\ln x}{a-x}\,dx = \pi a^{\mu-1}\left(\cot\pi\mu\ln a - \frac{\pi}{\sin^2\pi\mu}\right).$$