

**PROOF OF FORMULA 4.254.3**

$$\int_0^\infty \frac{\ln x \, dx}{(x^q - 1) x^p} = \frac{\pi^2}{\left(q \sin\left(\frac{p-1}{q}\pi\right)\right)^2}$$

The change of variables  $t = 1/x$  gives

$$\int_0^\infty \frac{\ln x \, dx}{(x^q - 1) x^p} = - \int_0^\infty \frac{t^{p+q-2} \ln t}{1 - t^q} dt.$$

The result now follows from entry 4.254.2:

$$\int_0^\infty \frac{x^{a-1} \ln x}{1 - x^b} dx = -\frac{\pi^2}{b^2 \sin^2(\pi a/b)}.$$