PROOF OF FORMULA 4.254.4

$$\int_0^1 \frac{x^{p-1}\,\ln x}{1+x^q}\,dx = \frac{1}{q^2}\beta'\left(\frac{p}{q}\right)$$

The change of variable $t = x^q$ gives

$$\int_0^1 \frac{x^{p-1}\,\ln x}{1+x^q}\,dx = \frac{1}{q^2}\int_0^1 \frac{t^{p/q-1}\,\ln t}{1+t}\,dt.$$

The beta function is defined by

$$\beta(a) = \int_0^1 \frac{t^{a-1}\,dt}{1+t}.$$

Therefore

$$\beta'(a) = \int_0^1 \frac{t^{a-1}\,\ln t}{1+t}\,dt.$$

The result follows from this expression.