PROOF OF FORMULA 4.257.3

$$\int_0^\infty \left(\frac{x^p}{q^{2p}+x^{2p}}\right)^r \ln\frac{x}{q}\,\frac{dx}{q^2+x^2} = 0$$

Let $x = qt$ to produce

$$\int_0^\infty \left(\frac{x^p}{q^{2p}+x^{2p}}\right)^r \ln\frac{x}{q}\,\frac{dx}{q^2+x^2} = q^{-rp-1}\int_0^\infty \left(\frac{t^p}{1+t^{2p}}\right)^r \frac{\ln t}{1+t^2}\,dt.$$

Split the integral over $[0,1]$ and $[1,\infty)$ and let $s = 1/t$ in the second interval. Then check that this integral becomes minus the one over $[0,1]$. Thus the total integral vanishes.