PROOF OF FORMULA 4.261.4

$$\int_0^\infty \frac{\ln^2 x \, dx}{(x-1)(x+a)} = \frac{\left[\pi^2 + \ln^2 a\right] \, \ln a}{3(a+1)}$$

The first paper in the series contains the proof of

$$\begin{aligned}\int_0^\infty \frac{\ln^{n-1} x \, dx}{(x-1)(x+a)} &= \frac{(-1)^n n! \, (1+(-1)^n) \, \zeta(n)}{n(1+a)} \\ &+ \frac{1}{n(1+a)} \left[\sum_{j=0}^{\lfloor n/2 \rfloor} \binom{n}{2j} (2^{2j}-2)(-1)^{j-1} B_{2j} \pi^{2j} \log^{n-2j} a \right].\end{aligned}$$

This entry corresponds to the case $n=3$.