PROOF OF FORMULA 4.261.5

$$\int_0^\infty \frac{\ln^2 x\, dx}{(1-x)^2} = \frac{2\pi^2}{3}$$

The change of variable $x \mapsto 1/x$ on the interval $[1, \infty)$ shows that

$$\int_0^\infty \frac{\ln^2 x\, dx}{(1-x)^2} = 2\int_0^1 \frac{\ln^2 x\, dx}{(1-x)^2}.$$

The expansion

$$\frac{1}{(1-x)^2} = \sum_{k=1}^\infty k x^{k-1}$$

and the evaluation

$$\int_0^1 x^{k-1} \ln^2 x\, dx = \frac{2}{k^3}$$

given in entry 4.261.18 produce the result.