PROOF OF FORMULA 4.262.6

$$\int_0^1 \frac{x^{2n}}{1-x^2} \ln^3 x \, dx = -\frac{\pi^4}{16} + 6 \sum_{k=0}^{n-1} \frac{1}{(2k+1)^4}$$

The change of variable $x = e^{-t}$ followed by $u = 2t$ give

$$\int_0^1 \frac{x^{2n}}{1-x^2} \ln^3 x \, dx = -\frac{1}{16} \int_0^\infty u^3 \frac{e^{-(n+1/2)u}}{1-e^{-u}} \, du.$$

Entry 3.411.6 gives

$$\int_0^1 \frac{x^{2n}}{1-x^2} \ln^3 x \, dx = -6 \sum_{k=n}^{\infty} \frac{1}{(2k+1)^4}.$$

This is the result.