PROOF OF FORMULA 4.265

$$\int_0^1 \frac{\ln^6 x\, dx}{1+x^2} = \frac{51\pi^7}{256}$$

The change of variables $x = \tan\varphi$ gives

$$\int_0^1 \frac{\ln^6 x\, dx}{1+x^2} = \int_0^{\pi/4} \ln^6 \tan\varphi\, d\varphi.$$

This is the case $n = 6$ in entry 4.227.4 and its value comes from $|E_7| = 61$.