PROOF OF FORMULA 4.266.2

$$\int_0^1 \frac{\ln^7 x\, dx}{1-x} = -\frac{8\pi^8}{15}$$

Entry 4.271.4 states that

$$\int_0^1 \frac{\ln^{p-1} x\, dx}{1-x} = (-1)^{p-1}\Gamma(p)\zeta(p).$$

Take $p = 8$ and use $\Gamma(8) = 5040$ and $\zeta(8) = \pi^8/9450$ to obtain the result.