

PROOF OF FORMULA 4.267.10

$$\int_0^1 \frac{x^{p-1} - x^{-p}}{(1+x) \ln x} dx = \frac{1}{2} \int_0^\infty \frac{x^{p-1} - x^{-p}}{(1+x) \ln x} dx = \ln \left(\tan \frac{\pi p}{2} \right)$$

To prove the identity of the two integrals split the second one over $[0, 1]$ and $[1, \infty)$ and let $x \mapsto 1/x$ in the second part.

The evaluation of the integral is done using entry 4.267.9 with $q = 1 - p$.