PROOF OF FORMULA 4.267.12

$$\int_0^1 \frac{x^p - x^q}{(1-ax)^n} \frac{dx}{x \ln x} = \ln \frac{p}{q} + \sum_{k=1}^{\infty} \binom{n+k-1}{k} a^k \ln \frac{p+k}{q+k}$$

The binomial theorem

$$(1-ax)^{-n} = \sum_{k=0}^{\infty} \binom{-n}{k} (-ax)^k$$

and the identity

$$\binom{-n}{k} (-a)^k = \binom{n+k-1}{k} a^k$$

give

$$\int_0^1 \frac{x^p - x^q}{(1-ax)^n} \frac{dx}{x \ln x} = \sum_{k=0}^{\infty} \binom{n+k-1}{k} a^k \int_0^1 \frac{x^{p+k-1} - x^{q+k-1}}{\ln x} \, dx.$$

The result now follows from entry 4.267.8

$$\int_0^1 \frac{x^{p-1} - x^{q-1}}{\ln x} \, dx = \ln \frac{p}{q}.$$