PROOF OF FORMULA 4.267.14

$$\int_0^1 \frac{(x^p - x^q)(1+x^{2n+1})}{(1+x)\,\ln x}\,dx = \ln\left[\frac{\Gamma\left(\frac{p}{2}+n+1\right)\Gamma\left(\frac{q+1}{2}+n\right)\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q}{2}\right)}{\Gamma\left(\frac{q}{2}+n+1\right)\Gamma\left(\frac{p+1}{2}+n\right)\Gamma\left(\frac{q+1}{2}\right)\Gamma\left(\frac{p}{2}\right)}\right]$$

Write the integral as

$$\int_0^1 \frac{x^{p-1}-x^{q-1}}{(1+x)\,\ln x}dx + \int_0^1 \frac{x^{(2n+p+1)-1}-x^{(2n+q+1)-1}}{(1+x)\,\ln x}dx,$$

and apply entry 4.267.9

$$\int_0^1 \frac{x^{a-1}-x^{b-1}}{(1+x)\,\ln x}\,dx = \ln\left(\frac{\Gamma\left(\frac{b}{2}\right)\Gamma\left(\frac{a+1}{2}\right)}{\Gamma\left(\frac{a}{2}\right)\Gamma\left(\frac{b+1}{2}\right)}\right)$$

to each one of them.