PROOF OF FORMULA 4.267.20

$$\int_0^1 \frac{(x^{p-1}-x^{q-1})}{(1-x^{2n})}\frac{(1-x^2)}{\ln x}\,dx = \ln\left[\frac{\Gamma\left(\frac{p+2}{2n}\right)\Gamma\left(\frac{q}{2n}\right)}{\Gamma\left(\frac{q+2}{2n}\right)\Gamma\left(\frac{p}{2n}\right)}\right]$$

The change of variables $t = x^{2n}$ gives

$$\int_0^1 \frac{(x^{p-1}-x^{q-1})}{(1-x^{2n})}\frac{(1-x^2)}{\ln x}\,dx = \int_0^1 \frac{(t^{p/2n-1}-t^{q/2n-1})}{(1-t)}\frac{(1-t^{1/2n})}{\ln t}\,dt.$$

The value of this integral appears in entry 4.267.15.