PROOF OF FORMULA 4.267.8

$$\int_0^1 \frac{x^{p-1}-x^{q-1}}{\ln x}\,dx = \ln\left(\frac{p}{q}\right)$$

The change of variables $x = e^{-t}$ gives

$$\int_0^1 \frac{x^{p-1}-x^{q-1}}{\ln x}\,dx = -\int_0^\infty \frac{e^{-pt}-e^{-qt}}{t}\,dt.$$

This is a Frullani type integral and its value, $\ln(p/q)$, was established in 3.434.2.