PROOF OF FORMULA 4.267.9

$$\int_0^1 \frac{x^{p-1}-x^{q-1}}{\ln x}\frac{dx}{1+x} = \ln\left(\frac{\Gamma(\frac{q}{2})\Gamma(\frac{p+1}{2})}{\Gamma(\frac{p}{2})\Gamma(\frac{q+1}{2})}\right)$$

The change of variables $x = e^{-t}$ gives

$$\int_0^1 \frac{x^{p-1}-x^{q-1}}{\ln x}\frac{dx}{1+x} = \int_0^\infty \frac{e^{-qt}-e^{-pt}}{1+e^{-t}}\,\frac{dt}{t}.$$

This last integral appears in entry 3.411.28.