PROOF OF FORMULA 4.269.3

$$\int_0^1 \sqrt{\ln \frac{1}{x}}\; x^{p-1}\, dx = \frac{1}{2}\sqrt{\frac{\pi}{p^3}}$$

Let $u = \ln 1/x = -\ln x$ to obtain

$$\int_0^1 \sqrt{\ln \frac{1}{x}}\; x^{p-1}\, dx = \int_0^\infty \sqrt{u}\, e^{-pu}\, du.$$

The change of variables $t = pu$ and the value

$$\int_0^\infty t^{1/2} e^{-t}\, dt = \Gamma\left(\tfrac{3}{2}\right) = \frac{\sqrt{\pi}}{2}$$

give the result.