PROOF OF FORMULA 4.269.4

$$\int_0^1 \frac{x^{p-1}\,dx}{\sqrt{\ln 1/x}} = \sqrt{\frac{\pi}{p}}$$

Let $u = \ln 1/x = -\ln x$ to obtain

$$\int_0^1 \frac{x^{p-1}\,dx}{\sqrt{\ln 1/x}} = \int_0^\infty \frac{e^{-pu}\,du}{\sqrt{u}}.$$

The change of variables $t = pu$ and the value

$$\int_0^\infty t^{-1/2} e^{-t}\,dt = \Gamma\left(\tfrac{1}{2}\right) = \sqrt{\pi}$$

gives the result.