

**PROOF OF FORMULA 4.269.7**

$$\int_u^v \frac{dx}{x\sqrt{\ln(x/u)\ln(v/x)}} = \pi$$

The change of variables  $t = x/u$  gives

$$\int_u^v \frac{dx}{x\sqrt{\ln(x/u)\ln(v/x)}} = \int_1^c \frac{dt}{t\sqrt{\ln t(\ln c - \ln t)}}$$

where  $c = v/u$ . Now let  $z = \ln t$  to obtain

$$\int_1^c \frac{dt}{t\sqrt{\ln t(\ln c - \ln t)}} = \int_0^a \frac{dz}{z(a-z)},$$

with  $a = \ln c$ . The change of variables  $z = a \sin^2 \varphi$  gives the result.