PROOF OF FORMULA 4.271.11

$$\int_0^1 (\ln x)^{2n-1} \frac{x\,dx}{1-x^2} = -\frac{1}{4n}|B_{2n}|\pi^{2n}$$

Let $t = x^2$ to obtain

$$\int_0^1 (\ln x)^{2n-1} \frac{x\,dx}{1-x^2} = \frac{1}{2^{2n}} \int_0^1 \frac{(\ln t)^{2n-1}}{1-t}\,dt.$$

The result now follows from entry 4.271.3:

$$\int_0^1 \frac{(\ln t)^{2n-1}}{1-t}\,dt = -\frac{2^{2n-2}}{n}|B_{2n}|\pi^{2n}.$$