

**PROOF OF FORMULA 4.271.14**

$$\int_0^\infty \frac{x^{\nu-1} \ln^n x dx}{x^2 + 2ax \cos t + a^2} dx = -\frac{\pi}{\sin t} \frac{d^n}{d\nu^n} \left[ a^{\nu-2} \frac{\sin(\nu-1)t}{\sin \pi\nu} \right]$$

Let

$$f(a) = \int_0^\infty \frac{x^{\nu-1} dx}{x^2 + 2ax \cos t + a^2}.$$

The scaling  $x = ay$  yields

$$f(a) = a^{\nu-2} \int_0^\infty \frac{y^{\nu-1} dy}{y^2 + 2y \cos t + 1}.$$

Factor

$$y^2 + 2y \cos t + 1 = (y + \alpha)(y + \beta)$$

with  $\alpha = -e^{-it}$  and  $\beta = -e^{it}$ .

Entry 3.223.1 gives

$$\int_0^\infty \frac{x^{\nu-1} dx}{(x + \alpha)(x + \beta)} = \frac{\pi}{\alpha - \beta} (\beta^{\nu-1} - \alpha^{\nu-1}) \operatorname{cosec}(\pi\nu).$$

The values of  $\alpha$  and  $\beta$  now give

$$\int_0^\infty \frac{x^{\nu-1} dx}{(x + \alpha)(x + \beta)} = -\frac{\pi}{\sin t} \frac{\sin((\nu-1)t)}{\sin \pi\nu}.$$

The result follows by differentiating  $n$  times with respect to  $\nu$ .