PROOF OF FORMULA 4.271.15

$$\int_0^1 \ln^n x \, \frac{x^{p-1}}{1-x^q} \, dx = -\frac{1}{q^{n+1}} \psi^{(n)}\left(\frac{p}{q}\right)$$

The change of variables $t = x^q$ yields

$$\int_0^1 \ln^n x \, \frac{x^{p-1}}{1-x^q} \, dx = \frac{1}{q^{n+1}} \int_0^1 \ln^n t \, \frac{t^{p/q-1}}{1-t} \, dt.$$

The result now follows by differentiating the integral representation

$$\psi(z) = \int_0^1 \frac{t^{z-1}-1}{t-1} \, dt - \gamma$$

n times with respect to z and then replacing $z = p/q$.