

PROOF OF FORMULA 4.272.15

$$\int_0^1 \left(\ln \frac{1}{x}\right)^n (1+x^q)^m x^{p-1} dx = n! \sum_{k=0}^m \binom{m}{k} \frac{1}{(p+kq)^{n+1}}$$

The change of variables $t = \ln 1/x$ yields

$$\int_0^1 \left(\ln \frac{1}{x}\right)^n (1+x^q)^m x^{p-1} dx = \int_0^\infty t^n e^{-pt} (1+e^{-qt})^m dt.$$

Expand the binomial term to obtain

$$\int_0^1 \left(\ln \frac{1}{x}\right)^n (1+x^q)^m x^{p-1} dx = \sum_{k=0}^m \binom{m}{k} \int_0^\infty t^n e^{-t(p+kq)} dt.$$

The result now follows from the change of variables $v = t(p+kq)$ and the value

$$\int_0^\infty v^n e^{-v} dv = \Gamma(n+1) = n!.$$