PROOF OF FORMULA 4.272.6

$$\int_0^1 \left(\ln \frac{1}{x}\right)^{\mu-1} x^{\nu-1} dx = \frac{\Gamma(\mu)}{\nu^{\mu}}$$

The change of variables $t = \ln(1/x)$ gives

$$\int_0^1 \left(\ln \frac{1}{x}\right)^{\mu-1} x^{\nu-1} dx = \int_0^{\infty} z^{\mu-1} e^{-\nu z}\, dz.$$

Now let $s = \nu z$ to obtain the result using the integral representation

$$\Gamma(a) = \int_0^{\infty} t^{a-1} e^{-t}\, dt.$$