

PROOF OF FORMULA 4.272.7

$$\int_0^1 \left(\ln \frac{1}{x}\right)^{n-1/2} x^{\nu-1} dx = \frac{(2n-1)!!}{(2\nu)^n} \sqrt{\frac{\pi}{\nu}}$$

Formula 4.272.6 states that

$$\int_0^1 \left(\ln \frac{1}{x}\right)^{\mu-1} x^{\nu-1} dx = \frac{1}{\nu^\mu} \Gamma(\mu).$$

The special value $\mu = n + \frac{1}{2}$ gives the result using

$$\Gamma\left(n + \frac{1}{2}\right) = \frac{\sqrt{\pi}(2n)!}{2^{2n}n!} = \frac{\sqrt{\pi}(2n-1)!!}{2^n}.$$