$$\int_0^1 \left(\frac{1}{\ln x} + \frac{1}{1-x}\right) \, dx = \gamma$$

Entry 4.281.4 states that

$$\psi(a) = \left(\frac{1}{\ln x} + \frac{x^{a-1}}{1-x}\right) \, dx.$$

Put a = 1 and use  $\psi(1) = -\gamma$ .