PROOF OF FORMULA 4.281.5

$$\int_0^1 \left(\frac{x^{p-1}}{\ln x} + \frac{x^{q-1}}{1-x} \right) \, dx = \ln p - \psi(q)$$

Write this as

$$\int_0^1 \left(\frac{x^{p-1}}{\ln x} + \frac{x^{q-1}}{1-x}\right) \, dx = \int_0^1 \left(\frac{1}{\ln x} + \frac{x^{q-1}}{1-x}\right) \, dx + \int_0^1 \frac{x^{p-1} - 1}{\ln x} \, dx.$$

The first integral is $-\psi(q)$ according to entry 4.281.4 and the second one is $\ln p$ according to 4.267.8.