PROOF OF FORMULA 4.283.4

$$\int_0^1 \left[\frac{1}{\ln^2 x} - \frac{x}{(1-x)^2} \right] dx = \gamma - \frac{1}{2}$$

Entry 4.281.1 states that

$$\int_0^1 \left[\frac{1}{\ln x} + \frac{1}{1-x} \right] dx = \gamma.$$

Integrate this formula by parts to obtain

$$\gamma = \lim_{x \to 1} x \left(\frac{1}{\ln x} + \frac{1}{1 - x} \right) + \int_0^1 \left[\frac{1}{\ln^2 x} - \frac{x}{(1 - x)^2} \right] dx.$$

Now check that the limit is $\frac{1}{2}$.