

PROOF OF FORMULA 4.291.1

$$\int_0^1 \frac{\ln(1+x)}{x} dx = \frac{\pi^2}{12}$$

Expand the integrand to obtain

$$\int_0^1 \frac{\ln(1+x)}{x} dx = \int_0^1 \sum_{k=0}^{\infty} \frac{(-1)^k x^k}{k+1} dx.$$

Therefore

$$\int_0^1 \frac{\ln(1+x)}{x} dx = \sum_{k=0}^{\infty} \frac{(-1)^k}{(k+1)^2}$$

The usual even-odd splitting gives the value $\pi^2/12$.