

PROOF OF FORMULA 4.292.2

$$\int_0^1 \frac{x \ln(1 \pm x)}{\sqrt{1-x^2}} dx = -1 \pm \frac{\pi}{2}$$

Integrate by parts to obtain

$$\int_0^1 \frac{x \ln(1 \pm x)}{\sqrt{1-x^2}} dx = \pm \int_0^1 \frac{\sqrt{1-x^2}}{1 \pm x} dx.$$

The change of variables $x = \sin t$ yields

$$\pm \int_0^1 \frac{\sqrt{1-x^2}}{1 \pm x} dx = \pm \int_0^{\pi/2} (1 \mp \sin t) dt.$$

The last integral is elementary and it gives the result.