PROOF OF FORMULA 4.293.4

$$\int_0^1 x^{2n-1} \ln(1+x)\, dx = \frac{1}{2n} \sum_{k=1}^{2n} \frac{(-1)^{k-1}}{k}$$

Integrate by parts to produce

$$\int_0^1 x^{2n-1} \ln(1+x)\, dx = \frac{\ln 2}{a+1} - \frac{1}{a+1} \int_0^1 \frac{x^{a+1}\, dx}{1+x}.$$

In the special case $a = 2n-1$ this gives

$$\int_0^1 x^{2n-1} \ln(1+x)\, dx = \frac{\ln 2}{2n} - \frac{1}{2n} \int_0^1 \frac{x^{2n}\, dx}{1+x}.$$

Now write

$$\frac{x^{2n}}{1+x} = \frac{x^{2n}-1}{1+x} + \frac{1}{1+x}$$

and one obtains

$$\int_0^1 x^{2n-1} \ln(1+x)\, dx = -\frac{1}{2n} \int_0^1 \frac{x^{2n}-1}{1+x}\, dx.$$

The result now comes from integrating the identity

$$\frac{x^{2n}-1}{1+x} = \sum_{k=0}^{2n-1} (-1)^{k-1} x^k.$$