PROOF OF FORMULA 4.293.8

$$\int_0^1 x^{\mu-1}\,\ln(1-x)\,dx = -\frac{1}{\mu}\left[\psi(\mu+1)+\gamma\right]$$

Entry 4.293.13 states that

$$\int_0^1 \frac{x^{\mu-1}\,\ln(1-x)}{(1-x)^{1-\nu}}\,dx = B(\mu,\nu)\left[\psi(\nu)-\psi(\mu+\nu)\right].$$

Now let $\nu = 1$ and use $B(\mu,1) = 1/\mu$ and $\psi(1) = -\gamma$ to obtain the result.