PROOF OF FORMULA 4.325.12

$$\int_0^1 \ln\ln(1/x) \, (\ln 1/x)^{\nu-1} \, x^{\mu-1} \, dx = \frac{\Gamma(\nu)}{\mu^\nu} \left[\psi(\nu) - \ln\mu\right]$$

The change of variables $t = \ln 1/x$ gives

$$\int_0^1 \ln\ln(1/x) \, (\ln 1/x)^{\nu-1} \, x^{\mu-1} \, dx = \int_0^\infty t^{\nu-1} e^{-\mu t} \, \ln t \, dt.$$

This appears as entry 4.352.1.