PROOF OF FORMULA 4.335.1

$$\int_0^\infty e^{-\mu x} \ln^2 x \, dx = \frac{1}{\mu} \left(\frac{\pi^2}{6} + (\gamma + \ln \mu)^2 \right)$$

Start with

$$\int_0^\infty x^a e^{-\mu x} \, dx = \mu^{-(a+1)} \Gamma(a+1)$$

and differentiate twice with respect to a to produce

$$\int_0^\infty x^a e^{-\mu x} \ln^2 x \, dx = \mu^{-(a+1)} \Gamma(a+1) \left[(-\ln \mu + \psi(a+1))^2 + \psi'(a+1) \right].$$

The result follows by choosing $a = 0$ and using the values

$$\psi(1) = -\gamma \text{ and } \psi'(1) = \pi^2/6.$$