PROOF OF FORMULA 4.335.2

$$\int_0^\infty e^{-x^2} \ln^2 x \, dx = \frac{\sqrt{\pi}}{16} \left(2(\gamma + 2\ln 2)^2 + \pi^2 \right)$$

Start with the identity

$$\int_0^\infty x^a e^{-x^2} \, dx = \frac{1}{2} \int_0^\infty t^{(a-1)/2} e^{-t} \, dt = \frac{1}{2} \Gamma\left(\frac{a+1}{2} \right).$$

Differentiate twice with respect to a to obtain

$$\int_0^\infty x^a e^{-x^2} \ln^2 x \, dx = \frac{1}{8} \Gamma\left(\frac{a+1}{2} \right) \left[\psi^2 \left(\frac{a+1}{2} \right) + \psi' \left(\frac{a+1}{2} \right) \right].$$

Now put $a = 0$ and use

$$\psi(1/2) = -\gamma - 2\ln 2 \text{ and } \psi'(1/2) = \pi^2/2$$

(the last value appears as entry 8.366) to obtain the result.