PROOF OF FORMULA 4.351.1

$$\int_0^1 (1-x)e^{-x}\ln x \, dx = \frac{1-e}{e}$$

Start with

$$\frac{d}{dx}(xe^{-x}) = (1-x)e^{-x},$$

so that

$$\int_{0}^{1} (1-x)e^{-x} \ln x \, dx = \int_{0}^{1} \ln x \, \frac{d}{dx} (xe^{-x}) \, dx.$$

The result follows from integration by parts.