PROOF OF FORMULA 4.352.1

$$\int_0^\infty x^{\nu-1}e^{-\mu x}\ln x\,dx = \frac{\Gamma(\nu)}{\mu^\nu}\left(\psi(\nu)-\ln\mu\right)$$

Let $t=\mu x$ to obtain

$$\int_0^\infty x^{\nu-1}e^{-\mu x}\ln x\,dx = \mu^{-\nu}\left(\int_0^\infty t^{\nu-1}e^{-t}\ln t\,dt - \ln\mu\int_0^\infty t^{\nu-1}e^{-t}\,dt\right).$$

The second integral is $\Gamma(\nu)$. The first one is $\Gamma'(\nu)$, obtained by differentiating

$$\Gamma(\nu)=\int_0^\infty t^{\nu-1}e^{-t}\,dt$$

with respect to the parameter ν. The result is now written in terms of the polygamma function $\psi(\nu)=\Gamma'(\mu)/\Gamma(\nu)$.