## PROOF OF FORMULA 4.352.3

$$\int_0^\infty x^{n-1/2} e^{-\mu x} \ln x \, dx = \frac{\sqrt{\pi}(2n-1)!!}{2^n \mu^{n+1/2}} \left[ 2 \left( 1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n-1} \right) - \gamma - \ln 4\mu \right]$$

Formula 4.352.1 states that

$$\int_0^\infty x^{\nu-1} e^{-\mu x} \ln x \, dx = \frac{\Gamma(\nu)}{\mu^\nu} \left[ \psi(\nu) - \ln \mu \right].$$

The special case $\nu = n + \frac{1}{2}$ gives

$$\int_0^\infty x^{n-1/2} e^{-\mu x} \ln x \, dx = \frac{\Gamma(n+1/2)}{\mu^{n+1/2}} \left[ \psi(n+1/2) - \ln \mu \right].$$

The duplication formula

$$\Gamma(2x) = \frac{2^{2x-1}}{\sqrt{\pi}} \Gamma(x) \Gamma(x+1/2)$$

give

$$\Gamma(n + \tfrac{1}{2}) = \frac{\sqrt{\pi}}{2^n} (2n-1)!!$$

and

$$\psi(n+\tfrac{1}{2}) = -\gamma + 2 \left[ 1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n-1} - \ln 2 \right].$$

Replace to obtain the result.