PROOF OF FORMULA 4.355.3

$$\int_0^\infty (\mu x^2 - n)x^{2n-1}e^{-\mu x^2} \ln x \, dx = \frac{(n-1)!}{4\mu^n}$$

In the proof of entry 4.355.1 the formula

$$\int_0^\infty x^m e^{-sx^b} \ln x \, dx = \frac{\Gamma(r)}{b^2 s^r} \left[\psi(r) - \ln s\right]$$

was established. Here $r = (m+1)/b$. Using this in the current problem, we have

$$\int_0^\infty (\mu x^2 - n)x^{2n-1}e^{-\mu x^2} \ln x \, dx = \mu\frac{\Gamma(n+1)}{4\mu^{n+1}} \left[\psi(n+1) - \ln \mu\right] - n\frac{\Gamma(n)}{4\mu^n} \left[\psi(n) - \ln \mu\right].$$

This reduces to the form stated here.