

A quartic integral.

Consider the integral

$$(1) \quad N_{0,4}(a, m) := \int_0^\infty \frac{dx}{(x^4 + 2ax^2 + 1)^{m+1}}$$

for $a \in \mathbb{R}$ and $m = 0, 1, \dots$

a) Find conditions in a that guarantee the convergence of the integral.

b) Use Mathematica to compute some values of $N_{0,4}(a, m)$. Give an empirical argument that suggests that the function

$$(2) \quad P_m(a) := \frac{1}{\pi} 2^{m+3/2} (a+1)^{m+1/2} N_{0,4}(a, m)$$

is interesting.

c) Give an experimental argument to show that $P_m(a)$ is a polynomial. Find its degree and give an explicit expression for the first few values of m .

d) Compute the Taylor series expansion of

$$(3) \quad h(a, c) := \sqrt{a + \sqrt{1+c}}$$

as a function of c , near $c = 0$. Explain your discovery.

Due on Wednesday 26th.