

Honors Calculus 131. Problem set 1.

Due August 29.

1) A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions a by b by cutting out equal squares of side x at each corner and then folding up the sides. Express the volume V of the box as a function of x . Give the domain of this function.

2) If $f(x) = 3x + 5$ and $g(x) = 15x + 8$. Find a function $h(x)$ such that $g = f \circ h$. Can this be done for any pair of linear functions f and g ?

3) Describe the graph of the function

$$f(x) = \frac{2x - 1}{3x - 4}.$$

Find the inverse of f . Be clear about domain and range. The set of functions of the form

$$g(x) = \frac{ax + b}{cx + d},$$

with $ad - bc \neq 0$ has very interesting properties. Functions of these form are called *Mobius transformations*. Prove that if you compute the composition of two Mobius transformations you get also a Mobius transformations. The same thing is true about inverses.

4) Let $f(x) = 7x^2 - 11x + 3$. Prove that if $|x|$ is sufficiently large, then

$$(7 - 10^{-5})x^2 \leq |f(x)| \leq (7 + 10^{-5})x^2.$$

You should be precise about how large $|x|$ has to be. Generalize this to the quadratic polynomial $g(x) = ax^2 + bx + c$.

5) Let $f(x) = x^2 - 3x + 1$. Prove that for x arbitrarily close to 1, the value of $f(x)$ is arbitrarily close to -1 . Your final answer should be stated like this: if x is at most a units away from 1, then $f(x)$ is b units away from -1 . Make your answer explicit for the case $a = 10^{-3}$.