

Honors Calculus 131. Problem set 3.

Due September 15.

1) Give a precise definition of

$$\lim_{x \rightarrow a} f(x) = +\infty$$

and use it to prove that

$$\lim_{x \rightarrow -1} \frac{2}{(x+1)^4} = +\infty.$$

2) Prove that

$$\lim_{x \rightarrow 1} \frac{2x+3}{3x+5} = \frac{5}{8}.$$

3) Let $f(x) = x^3 + x \sin(1/x)$. Prove that there is a number c such that $f(c) = 1000$. Can you estimate the value of c ?

4) Prove that $g(x) = |x|$ is a continuous function of x .

5) Show that if f is a continuous function, then so is $|f|$.

6) Suppose f is a function with the property $|f(x)| \leq x^2$ for all $x \in \mathbb{R}$. Show that $f(0) = 0$. Prove that the derivative exists at $x = 0$ and that $f'(0) = 0$. Does the result hold if you replace x^2 by $x^{1.9}$? How much smaller can you make this exponent?

7) Find the perimeter of a regular n -gon inscribed in a circle of radius r . Describe what happens as $n \rightarrow \infty$.