

Honors Calculus 131. Problem set 4.

Due September 29.

- 1) If a and b are positive numbers, find the maximum value of $f(x) = x^a(1-x)^b$ for $0 \leq x \leq 1$.
- 2) A cubic polynomial is a function of the form $f(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$. Count the number of critical points and give one example of each type.
- 3) Show that the equation $4x^5 + x^3 + 2x + 1 = 0$ has exactly one real root.
- 4) Prove that $\sqrt{1+x} < 1 + x/2$ for $x > 0$.
- 5) If $a_1 < a_2 < \cdots < a_n$, find the minimum value of the

$$f(x) = \sum_{i=1}^n (x - a_i)^2.$$

- 6) Let $a > 0$. Find the maximum value of the function

$$f(x) = \frac{1}{1+|x|} + \frac{1}{1+|x-a|}.$$

- 7) Prove that $\frac{1}{9} < \sqrt{66} - 8 < \frac{1}{8}$ without computing the square root.
- 8) Suppose f is a function that satisfies $f'(x) = 1/x$ for all $x > 0$ and $f(1) = 0$. We will prove later that $f(x) = \ln x$. Prove that $f(xy) = f(x) + f(y)$ for all $x, y > 0$.
Hint. Define the function g by $g(x) = f(xy)$ with y kept fixed. Now compute the derivative of g .