Honors Calculus 131. Problem set 5.
Due October 13.

1) A right circular cylinder is inscribed in a cone with height h and base radius r. Find the largest possible volume of such a cylinder.

2) Find the point on the line $x+y = 100$ that is closest to the ellipse $x^2/4+y^2/9 = 1$.

3) Two hallways, of width a and b, meet at right angles. What is the greatest possible length of a ladder which can be carried horizontally around the corner?

4) a) Suppose that the polynomial function $f(x) = x^n + a_{n-1}x^{n-1} + \cdots + a_0$ has exactly k critical points and $f''(x) \neq 0$ for all critical points. Prove that $n - k$ is odd.
b) For each n and k such that $n - k$ is odd, prove that there is a polynomial of degree n with k critical points.
c) Suppose that the polynomial f has k_1 local maximum points and k_2 local minimum points. Show that $k_2 = k_1 + 1$ if n is even and $k_2 = k_1$ if n is odd.
d) Assume that the conclusions of part c) hold. Prove that there is a polynomial f that has k_1 local maximum points and k_2 local minimum points. **Hint**. Pick numbers $a_1 < a_2 < \cdots < a_{k_1+k_2}$ and try

$$f'(x) = (x - a_1)(x - a_2)\cdots(x - a_{k_1+k_2}) \times (1 + x^2)^r$$

for some number r.

5) Apply Newton's method to the equation $1/x - a = 0$ to derive the recurrence

$$x_{n+1} = 2x_n - ax_n^2.$$

This shows you how to compute divisions.