

Honors Calculus 131. Problem set 8.

1) Find and prove the following limits

a) $\lim_{n \rightarrow \infty} \frac{n}{n+3}$

b) $\lim_{n \rightarrow \infty} \sqrt{n^2 + n} - \sqrt{n}$

c) $\lim_{n \rightarrow \infty} \sqrt[n]{a}$ for $a > 0$

d) $\lim_{n \rightarrow \infty} \frac{n!}{n^n}$

e) $\lim_{n \rightarrow \infty} (a^n + b^n)^{1/n}$

2) Let $0 < a_1 < b_1$ and define

$$a_{n+1} = \sqrt{a_n b_n}, \quad b_{n+1} = \frac{a_n + b_n}{2}$$

Prove that the sequences $\{a_n\}$ and $\{b_n\}$ converge to the same limit. **Hint.** Try to understand the quantity $a_{n+1} - b_{n+1}$.

3) Find the limit $\lim_{n \rightarrow \infty} n c^n$ for $|c| < 1$.

4) Evaluate

$$\lim_{n \rightarrow \infty} \frac{1}{n^{p+1}} \sum_{k=1}^n k^p$$