Honors Calculus 131. Problem set 9.

1) Determine whether the following sequences converge. If it converges, find its limit.
a) $a_n = \frac{n^3}{n^3+1}$
b) $a_n = n^2 e^{-n}$
c) $a_n = \tan^{-1}(2n)$

2) The Fibonacci sequence is defined recursively by $F_n = F_{n-1} + F_{n-2}$, starting with $F_0 = F_1 = 1$. Discuss the sequence

$$a_n = \frac{F_n}{F_{n-1}}$$

and decide whether it converges or not.

3) Prove that if $\lim_{n\to\infty} a_n = L$, then $\lim_{n\to\infty} a_{2n} = L$ and $\lim_{n\to\infty} a_{2n+1} = L$. Converesely, assume that this last two limits exist and they are equal. Prove that $\lim_{n\to\infty} a_n$ exists.

4) Determine whether the following series are convergent or divergent.
a) $\sum_{n=1}^{\infty} \frac{1+2^n}{3^n}$,
b) $\sum_{n=1}^{\infty} \frac{n^2}{n^2+1}$,
c) $\sum_{n=1}^{\infty} \left(e^{1/n} - e^{1/(n+1)}\right)$.

5) Solve problesm 63, 64 and 76 from Section 11.2 in the book.

6) Define

$$t_n = 1 + \tfrac{1}{2} + \tfrac{1}{3} + \cdots + \tfrac{1}{n}.$$

From an interpretation of t_n as an area, show that $t_n > 0$. Check that

$$t_n - t_{n+1} = (\ln(n+1) - \ln n) - \frac{1}{n+1}$$

and now interpret this expression as a difference of two areas to conclude that $t_n - t_{n+1} > 0$. Conclude that t_n is an decreasing sequence. Conclude that $\lim_{n\to\infty} t_n$ exists. This limit is called *Euler's constant*. There is a great book about it, called **GAMMA**. You should read it in your spare time.

7) Determine if the following series converge of diverge:
a) $\sum \frac{\tan^{-1} n}{n^a}$ where $a \in \mathbb{R}$.
b) $\sum \frac{1}{n^p \ln n}$ where $p \in \mathbb{R}$.
c) $\sum \frac{(n!)^2}{(kn)!}$ where k is a integer parameter.

8) Find the Taylor series of the function $f(x) = x/(1-x-x^2)$. Discuss its radius of convergence.