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Experimental mathematics in action. (EN)
[Book] Wellesley, MA: A K Peters (ISBN 978-1-56881-271-7/hbk). xii, 322 p. 49.00(2007).

The book is written by experts in experimental mathematics. It may be viewed as both an excellent textbook for beginners in the subject and a source book for experienced mathematicians. The introductory part (Chapter 1) presents the philosophy of the subject while the contents (Chapters 2–8) gives an overview of mathematical methods and computational tools applied to various problems in analysis, number theory, mathematical physics, and probability theory. An important ingredient of the book is the remarkable collection of exercises (Chapter 9).
In spite of its warlike title, "Experimental mathematics in action", the book is quite peaceful. It is very lovely reading for those who look for challenging unsolved problems, such as

$$\int_{\pi/3}^{\pi/2} \log\left|\frac{\tan t+\sqrt{7}}{\tan t-\sqrt{7}}\right| \mathrm{d}t \overset{?}{=} \frac{7\sqrt{7}}{24}\sum_{n=0}^{\infty}\left(\frac{1}{(7n+1)^2}+\frac{1}{(7n+2)^2}-\frac{1}{(7n+3)^2}\right.$$
$$\left.+\frac{1}{(7n+4)^2}-\frac{1}{(7n+5)^2}-\frac{1}{(7n+6)^2}\right),$$
$$\frac{128\sqrt{5}}{\pi^2} \overset{?}{=} \sum_{n=0}^{\infty}\frac{(\frac{1}{2})_n(\frac{1}{3})_n(\frac{2}{3})_n(\frac{1}{6})_n(\frac{5}{6})_n}{n!^5}(5418n^2+693n+29)\frac{(-1)^n}{80^{3n}},$$

or the question whether, for a given positive integer m, the expression

$$(m+l)(m-l+1)b_{l-1,m}^2 + l(l+1)b_{l,m}^2 - l(2m+1)b_{l-1,m}b_{l,m}$$

involving the binomial sums $b_{l,m} = \sum_{k=l}^{m} 2^k \binom{2m-2k}{m-k}\binom{m+k}{m}\binom{k}{l}$ attains its minimum at $l=m$. These problems were discovered experimentally and for the moment it is not clear which kind of solutions will appear first, computer-assisted or purely analytical (i.e., 'classical'). In the book under review, the reader will find references to both ways of resolving similar (but already solved) problems.
Among the drawbacks of the book, I would mention the authors' forgetting at some places to credit other people's contributions (even when the authors are aware of them). An example is identity (3.29),

$$\sum_{k=1}^{\infty}\frac{1}{k^3(1-x^4/k^4)} = \frac{5}{2}\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k^3\binom{2k}{k}(1-x^4/k^4)}\sum_{m=1}^{k-1}\frac{1+4x^4/m^4}{1-x^4/m^4},$$

experimentally discovered in [J. M. Borwein and D. M. Bradley, Exp. Math. **6**:3 (1997), 181–194. Zbl 0887.11037]. There are no other references besides this one; in particular, a reference to the first proof in [G. Almkvist and A. Granville, Exp. Math. **8**:2 (1999), 197–203. Zbl 0976.11035] is not given. Also omitted is an identity containing as particular cases both Koecher's identity (3.28) and the above-mentioned (3.29), namely, the generating series

$$\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}\binom{r+s}{r}\zeta(2r+4s+3)a^{2r}b^{4s}=\sum_{n=1}^{\infty}\frac{n}{n^4-a^2n^2-b^4}$$
$$=\frac{1}{2}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{\binom{2n}{n}n}\,\frac{5n^2-a^2}{n^4-a^2n^2-b^4}\prod_{l=1}^{n-1}\frac{(l^2-a^2)^2+4b^4}{l^4-a^2l^2-b^4},$$

originally conjectured by H. Cohen and proved recently by T. Rivoal [Exp. Math. **13**:4 (2004), 503–508. Zbl pre02150497].

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Primary Classification:
65-04 - Explicit machine computation and programs not the theory of computation or programming

Secondary Classification:
00A05 - General mathematics
11-04 - Explicit machine computation and programs not the theory of computation or programming
11Y35 - Analytic computations
11Y60 - Evaluation of constants
33Fxx - Computational aspects
65-05 - Experimental work
65B10 - Summation of series
65Q05 - Difference and functional equations, recurrence relations
65Y20 - Complexity and performance of numerical algorithms
65Z05 - Applications to physics
68W30 - Symbolic computation and algebraic computation

Keywords: *experimental number theory; experimental analysis; experimental probability theory; mathematica physics; symbolic computation*