Test 1. Practice problems.

These are practice problems for the first test.

After you work on them, if you want to discuss any of these problems or want to check your answer, you can email me at

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(1) Use the method of Riemann sums to evaluate the area of the triangle formed by the x-axis, the line $x = 3$ and the line $y = 2x$.
(2) Find a lower Riemann sums for the integral of $f(x) = 1 - x^2$ from $x = -2$ to $x = 4$ using six internal points.
(3) Write the area under the graph of $y = \sqrt{x}$ between $x = 1$ and $x = 4$ as the limit of a Riemann sum. Do not evaluate.
(4)
$$\text{If } \int_2^8 f(x)dx = 12, \text{ and } \int_2^4 f(x)dx = 5, \text{ find } \int_2^4 f(2x)dx.$$
(5) Differentiate the function
$$f(x) = \int_{2x}^{3} \frac{t^2 + t}{t^3 + 1}\, dt.$$
(6) Differentiate the function
$$g(x) = \int_{-2\sqrt{x}}^{x^2+x} \frac{1 + t^2 + t}{t^4 + 5}\, dt.$$
(7) Prove that
$$\int_0^1 x^a (1 - x)^b \, dx = \int_0^1 (1 - x)^a x^b \, dx.$$
Evaluate the integral for $a = 1$ and $b = 2$ and also for $a = b = 1/2$.
(8) Prove that
$$\frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} < \ln n < 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n-1}$$
(9) Find the limit
$$\lim_{h \to 0} \frac{1}{h} \int_2^{2+h} \sqrt{1 + t^3}\, dt.$$
Hint. Do not try to evaluate the integral.
(10) Find a number b such that the line $y = b$ divides the region bounded by the curves $= x^2$ and $y = 4$ into two regions with equal area.
(11) Find the area bounded by the curves $y = x^2 - 2$, $y = e^x$ and the lines $x = -1$ and $x = 1$.
(12) Find the area of the region bounded by the parabola $y = x^2$, the tangent line to this parabola at $x = 1$, and the x-axis.
(13) What is the volume of coffee inside a spherical container of radius 10, if the liquid is at height 2?

(14) Express the volume of the solid obtained by rotating a circle of radius a centered at the point $(0, -2a)$ about the x-axis.

(15) Evaluate

$$\int x^2 e^{-x}\,dx \quad \int x^2 \sin x\,dx \quad \int \sqrt{t}\ln(2t)\,dt \quad \int \frac{dx}{\cos x - 1}$$

$$\int \frac{t^2}{t^2+1}\,dt \quad \int \sqrt{4x^2+5x+1}\,dx \quad \int \cos^2 x \sin^2 x\,dx \quad \int \frac{t^5\,dt}{\sqrt{t^2+2}}$$

(16) Evaluate $\int \ln(x^2+3x+3)\,dx$

(17) If f is a quadratic function of x such that $f(0) = 1$ and

$$\int \frac{f(x)\,dx}{x^2\,(x+1)^2}\,dx$$

is a rational function of x, find the value of $f'(0)$.

(18) Find the length of the curve

$$y = \frac{x^4}{16} + \frac{1}{2x^2}$$

from $x = 1$ to $x = 2$. Find the area of the surface obtained by rotating the curve above about the x-axis.