

**Instructor: A.D.Wentzell**

The general spirit of the course: probability theory based on the theory of measure and integration. The students need to have had some exposure to measure theory – though they are not supposed to know everything in it by heart. The results in the theory of measure and integration will be formulated in the precise form in which they will be used every time they are needed, but without any proofs (the proofs and the material around the results mentioned can be found in books on Analysis, e. g., Kolmogorov and Fomin’s book “Introductory Real Analysis”).

The topics to be touched upon in the course: The axioms of probability theory, and measure theory; random variables and their distributions; Lebesgue integral; operations on distributions, including mixtures of distributions; distribution functions; expectations; product measures and independent random variables; the zero-one law; convergence of sequences of random variables, laws of large numbers; characteristic functions; weak convergence of distributions; limit theorems via characteristic functions; the central limit theorem; conditional probabilities and Markov chains; time-homogeneous Markov chains; using  $\sigma$ -algebras to characterise dependence: 1) “weak dependence” conditions, and 2) conditional expectations and conditional probabilities with respect to  $\sigma$ -algebras; introduction to Markov processes; the Wiener process.

I won’t give lectures based on any textbook; but I hope to be able to provide some lecture notes (accessible through my home page).