MATH 777 MARKOV STOCHASTIC PROCESSES SPRING 2009

Syllabus.

Instructor:

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Lectures:

Mondays, Wednesdays, Fridays 9 – 9:50 in Boggs 240.

Office hours:

Mondays, Wednesdays, Fridays 10 – 10:50, or by appointment

The material expected to be covered in the course:

1. The theme of Markov processes was started in my previous-semester course, as well as the basic concepts of probability theory (including conditional expectations and conditional probabilities with respect to σ-algebras); about 38 lecture notes for this course are available on my web page `http://www.math.tulane.edu/~wentzell`. I am going to refer to them as **2008.1**, or **2008.2**, etc.

2. Discrete Markov chains (partially covered in the fall 2008 course). The state space. Transition probabilities, the initial distribution. Stochastic matrices. Transition probabilities in k steps; the transition matrix is equal to the k-th power of the one-step transition matrix.

Ergodicity; the equations for the limiting probabilities.

Classification of states. Classes of communicating states; periodic states. Recurrent and transient states.

Algebraic equations for the expectation of the first time of leaving the region, and other expectations associated with this random time.

Particular case: branching processes.

3. Continuous-time Markov chains. Transition probabilities. The Chapman – Kolmogorov equations; the same equations in the matrix form.

Jump times. Exit rates, transition rates.

The first jump time from a state x (non-random) has an exponential distribution with parameter equal to the exit rate from this state. The first jump time and the state to which the chain jumps are independent. Relation of their distributions to the transition rates. The times between the jumps and their distribution.

Kolmogorov differential equations. Forward Kolmogorov equations; backward Kolmogorov equations. Their matrix form.

The problem of ergodicity. The algebraic equations for the stationary (steady-state) distribution.

4. Examples of continuous-time Markov chains. The Poisson process. Birth-and-death processes. Elements of queuing theory.

5. Martingales. Stopping times.
Applications to discrete Markov chains and to continuous-time Markov chains.

6. Continuous-space Markov processes. The Brownian motion.
The "quadratic variation" of the Brownian motion (the limit of the sum of the squares of its increments). The differential operator associated with the Brownian motion.

7. Linear operators associated with a Markov process. The infinitesimal operator. The Kolmogorov equations.

8. Stochastic integrals. Stochastic differentials. Itô's formula.
Stochastic differential equations. Diffusion processes.
The infinitesimal operator of a diffusion process. Differential equations associated with diffusion processes.

I hope to be able to provide the students with lecture notes for the present course. These lecture notes will be referred to as **Lecture 1**, or **Lecture 2**, etc.

I'll try to invent problems on the material covered for you to solve.