Math 603, Summer 2006, Homework Set 6

Due Thursday, June 8 th

1. Consider a birth and death chain on $\{0, 1, \ldots, d\}$ where d may take the value ∞. For $x \in \mathcal{I}$, the transition function takes the form

$$P(x,y) = \begin{cases} q_x & : \quad y = x-1 \\ r_x & : \quad y = x \\ p_x & : \quad y = x+1 \end{cases}$$

where $q_x + r_x + p_x = 1$, $q_0 = 0$, and $p_d = 0$ if $|\mathcal{I}| < \infty$. Set $\gamma_0 = 1$, and for $0 < i < d$, set

$$\gamma_i = \frac{q_1 \cdots q_i}{p_1 \cdots p_i}$$

Let $a, b \in \mathcal{I}$ with $a < b$. Using the work we discussed in class, show that for all x such that $a < x < b$,

$$P_x(T_b < T_a) = \frac{\sum_{i=a}^{x-1} \gamma_i}{\sum_{i=a}^{b-1} \gamma_i}$$

2. A gambler playing roulette makes a series of one dollar bets. He has respective probabilities 9/19 and 10/19 of winning and losing each bet. The gambler decides to quit playing as soon as he either is one dollar ahead or has lost his initial capital of $1000.
a). Find the probability that when he quits playing he will have lost $1000.
b). Find his expected loss.

3. Consider an irreducible birth and death chain on the nonnegative integers such that $p_x \leq q_x$ for all $x \geq 1$. Show that the chain is recurrent.

4. Consider an irreducible birth and death chain on the nonnegative integers such that $q_x = \frac{x^2}{2x^2+2x+1}$, $p_x = \frac{(x+1)^2}{2x^2+2x+1}$ for all $x \geq 1$. Is the chain recurrent? Prove or disprove your answer.